A New Approach to Structural Equivalence: Places and Networks of Places as Tools for Sociological Theory

Narciso Pizarro*
Universidad Complutense de Madrid

Introduction

The recent discussions of Simmel's contribution to structural social sciences (BREIGER: 1990), have not yet completely reached the mathematical modelling of networks. In Simmel's terms, individual identities are considered as a property of the network of relations between membership groups. This follows the basic idea of all structural analysis that entities should be defined as a result of the relational systems and not independently from them. However, there is then a gap between the actual level of theoretical understanding of structural concepts and the basic mathematical modelling. This gap appears in social network analysis when social networks are defined as a set of individuals as nodes and a set of inter-individual relations defined as contingent phenomena concerning individual nodes while looking for social structure by these means. Many difficulties arise in concrete, empirical research based on this kind of data.

The first of these difficulties is how to compare the structure of a social group in two different times, separated by a time interval of a few decades, if the social networks we use to represent social structure have as nodes concrete individuals, identified by their names. Individuals identified in the first instance will not appear in the second, many years after: they will be dead[1]. New individuals, not appearing in the first network, will be the nodes of the

* The present article owes to FRANÇOIS LORRAIN the elegance of the mathematical formalization and to DOUGLAS R. WHITE the careful edition that helped the author to suppress many of the initial errors. All the remaining defaults are entirely author´s responsability.

[1] But "dead" does not mean necessarily biological death: comparing two networks built from data separated in time by, for example, only five years, individuals of the first disappear in the second, as new ones appears that were not in the first. Individuals who do not appear in the second network are not necessarily dead nor are those who now appear new born: what happens is simply that individuals are not forever in a given social position. The effects of time on networks whose nodes are concrete individuals do not reduce to the case of generational replacement: social time has different cycles that we are starting to study properly [Moody,J. And White, D.R., 1999]

second. The nodes of the two networks being different, the comparison becomes impossible but, nevertheless, we often practically know that social structures are comparable.

Second, when the time interval separating two empirical descriptions of social facts is long enough, we will find that the identity of social subgroups (and in some cases, of social institutions related to them) has also changed. The social relations defined by and associated with membership will then not be comparable.

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This kind of practical and empirical problems are solved when the operational definition of social networks takes into account the theoretical conceptions of structural sociology.

The concept of places and of the networks of places presented here may contribute to solve many of the problems previously mentioned. We define the place of an individual as the subset of groups to which he belongs, contained in the set of all the groups to which all the individuals in the network belongs. So defined, *a place is a relational entity* that expresses the intrinsic relational nature of individual identity, as well as, at the same time, the relational character of group identity. The algebra of networks of places shown how initially defined nuclear groups can be derived from their relations in a network of places. We will see also that places can be viewed as defining sets of structurally equivalent individuals.

2.- What do we mean by individuals?

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When we say: let's consider two individuals, A and B. How do we know that they are different from each other ? The answer to this question seems so obvious that we never discuss it. Nevertheless, individual identity is not a simple matter.

The attributes used to socially differentiate individuals are mainly social. The individual's name is a social institution, regulated by custom, and by civil law in contemporary states. Names are inherited, and so, related to parenthood systems, the primordial social institution. Names means, mainly " Individual A is a member of a family, X". She or he is a daughter/son of a particular couple of other individuals.

Other attributes of individuals, used to identify them, are social institutions as well: professions can be viewed as the names socially given to individuals who belong to specific social groups. To say “A is a lawyer” means also “A is member of the Bar”. Most of the characteristics of individuals, as well as professions, with the main exception of physical attributes of the body, are socially constructed categories, related to social groups with a certain level of institutionalisation.

Taking this point of view, it is possible to *define individual identity as a set of membership relations*. This definition of identity is, as we will see later, an operational concept that can be used as a powerful tool in structural social sciences. As Breiger (1990) shows, Simmel’s ideas on the intersection of social circles (1927) can be understood as a logic of identity deriving from multiple memberships.

3.- What do we mean by social relations?

The expression *social relation* is, at least, as seemingly obvious as the meaning of the term *individual*: social relations are relations between individuals. Social sciences take them as facts because they seem to appear like facts in individual consciousness.

Social research constructs data sets of social relations asking individuals about their social relations, and noting their answers in a more or less careful manner. Registered answers are very heterogeneous: some of them refer to emotions, other to values and attitudes, both merely psychological entities. But some species of answers refer also to objective social process, in a way that, because of their psychological nature, it is not always easy to relate to the social facts we are looking for.

Since Nadel (), we know that not all of the individual’s interactions have to be studied by social sciences. Some interactions are merely random events, never reproduced, that can not or should not be confused with social relations: the expression social relation should be used only for patterns of interaction that are *independent from the particular individuals involved* in them. Those patterns should be also *regularly existing in time, over periods longer than individual’s human life*.

Some psychological entities noted in empirical research respect the preceding criteria, but many don’t. It is not easy, from a strictly empirical standpoint, to distinguish one from another in what concerns their individual character or their properties in time.

But not all of them are *only* psychological entities: in the individual's answers, we get subjective references to social facts that can be observed in the outside world, out of the individual's consciousness.

Traces of many social relations are kept in *archives, where their inscription is institutionally produced*: inscription is a social process which depends upon the relation's compliance with rules and norms. And it implies the acknowledgement of the relation by a social group, in such a way that it is no longer a inter-subjective matter, but a socially regulated social relation: a marriage is more than an inter-individual affair as the inter subjective relation become properly published, recognised and deserving an inscription in the collective memory.

4.- Membership relations and their sociological meaning

The individual's membership to a particular social group is a very general kind of social relation, embedded in our ways of talking about ourselves and about our relations with others. What has being said before about individual identity is a very important part of the sociological meaning of membership relations but it is not the whole: it remains another aspect.

In addition to its contribution to the definition of identity, *membership determines also the nature and contents of inter-individual interaction*. It is because subject X belongs to the group of physicians that he can have a particular relation with subject Y, involving the possibility for X of modifying Y's behaviour (eat or drink a particular substance), as Y recognises X's membership to the group of doctors and himself as needing a doctor's care.

The concept of role is not an adequate instrument to capture this very important aspect of social structure: the social existence of a doctor has more contents than those captured by the mention of an individual in a doctor's role. As well, relations between corporation board members cannot be defined and understood from their mutual interaction, forgetting the determinant character of their common membership.

Common memberships explain the content of relations better than data proceeding from subjective descriptions, and it is more so when the relations under study are, as they should be in social sciences, un-individualised and long-lasting types of interaction. Only for the types of relations that Simmel considers as merely inter-subjective, do membership's determinations lose their explanatory power.

5.- The concept of place: networks of places

The preceding considerations came, of course, after the need for a new concept became evident, one more adequate to capture the un-individualised and time-regular character of social relations, and as it became evident how to define it. Dealing with data about multiple memberships of thousand of individuals over decades or long periods of time, it is clear that the following definition is, for the moment being, a convenient research tool.

If we have a set of individuals

$$I = \{i_1, i_2, i_3, ...\}$$

belonging each one to one or more ***socially*** (not only analytically) defined sets of individuals, here called *institutions* to stress with the word the *socially built* character of the set and noted

$$E = \{e_1, e_2, e_3, ...\}$$

we will define a *place P* as a subset of *E* such that *at least one* of the individuals members of *I* belongs to every one and only to the institutions included in the subset *P*. That is to say

$$P_i = \{e_j \in E : I_i \in e_j\}$$

So defined, *places are subsets of E*, independent from individuals not only because we can find many individuals having the same place, but also because, thinking in time, an individual's place can last longer than the individual himself, as others one can occupy a particular place after the first one disappears.

Also, social structure can be defined as a network of places, using the following straightforward definition of a relation between two places:

Definition 2: Two places $\boldsymbol{P_i}$ and $\boldsymbol{P_j}$ are in a relation $\boldsymbol{R}$ if $\boldsymbol{P_i} \cap \boldsymbol{P_j} \neq \emptyset$

The set $\boldsymbol{P}$ of all the places defined in $\boldsymbol{E}$ and the set $\boldsymbol{R}$ of their relations constitutes a ***network of places.***

6.- Structural Equivalence and Networks of Places

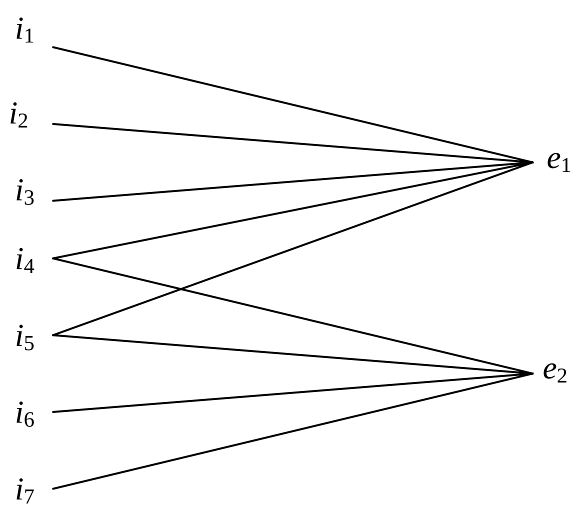
In a network of individuals, we can identify sets of structurally equivalent individuals (LORRAIN & WHITE, 1971). We can look at structurally equivalent individuals in a network of membership relations. We have a set of individuals

$$I = \{i_1, i_2, i_3, \ldots\}$$

and a set of institutions

$$E = \{e_1, e_2, e_3, \ldots\}$$

each individual is a member of a certain subset of institutions. For example, if I 's set cardinal's is seven, where E had two elements, we could have the following membership relations

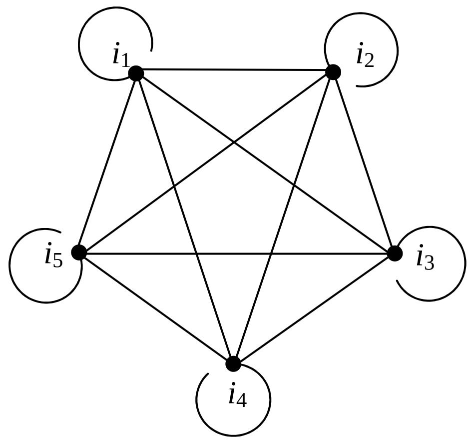


We consider an individual i_r to be linked to an individual i_s through a link e_k if both individuals are members of e_k:

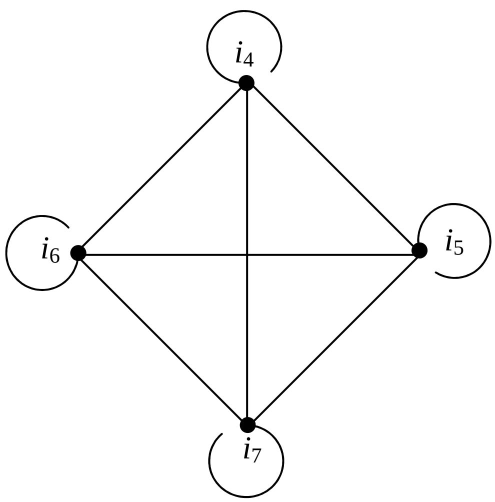
$$i_r \xrightarrow{e_k} i_s \text{ if } i_r, i_s \in e_k$$

In our example, we will have the following

Links of type e_1 between individuals



Links of type e_2 between individuals



In this network, *two individuals are structurally equivalent if they are both members of, precisely, the same institutions*. In our example, individuals i_1, i_2 and i_3 are all members of the same institution e_1 and to no other; consequently

i_1, i_2 and i_3 are structurally equivalent.

Individuals i_4 and i_5 are both members of institution e_1 and e_2 and to no other , and so

i_4 and i_5 are structurally equivalent

Similarly, i_6 and i_7 are members of e_2 and they do not belong to any other institution, thus

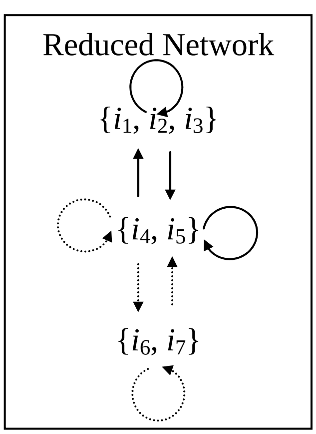
i_6 and i_7 are structurally equivalent

Two individuals are structurally equivalent if the occupy the same *place*, in the sense previously defined. *Each class of equivalent individuals corresponds to a place*, that is to say, to the set of institutions that defines the place itself. In our example, we have three classes of equivalence of individuals corresponding to the following places:

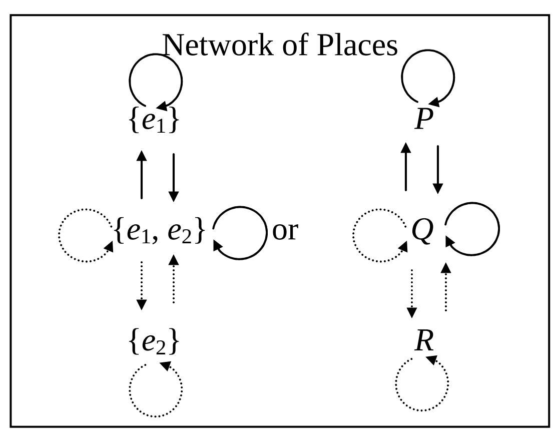
Equivalence classes	*Corresponding places*
$\{i_1, i_2, i_3\}$	$\{e_1\}$
$\{i_4, i_5\}$	$\{e_1, e_2\}$
$\{i_6, i_7\}$	$\{e_2\}$

Each class of equivalence of an individual corresponds to an unique place and, reciprocally, each place defines an unique equivalence class of individuals. The expressions *equivalence class of individuals* and *place* are practically synonymous.

If structurally equivalent individuals are identified, a reduced network is obtained, where points are equivalence classes, that is to say places. In our example the reduced network has the following appearance, where links e_1 are represented by continuous arcs and those of e_2 type, by dotted arcs.



or the equivalent



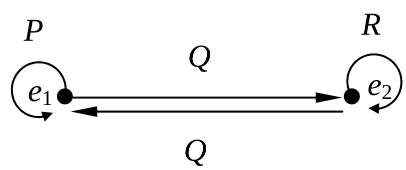
A place X is linked to a place Y by a link e_k if individuals belonging to X and to Y belongs all of them to institution e_k – that is to say, if institution e_k belongs simultaneously to the set of institutions of the place X and to the one of place Y:

$$X \xrightarrow{e_k} Y \text{ if } X, Y \supseteq e_k$$

We saw how the network of individuals linked by institutions can be reduced to a *network P of places linked by institutions*. It is possible, also, to define a dual network: a *network P* of institutions linked by places*. In this dual network an institution e_k is linked to an institution e_l by a link P if these institutions belongs both to the place P:

$$e_k \xrightarrow{p_r} e_l \text{ if } e_k, e_l \in P$$

In our example, the dual network P^* is the following:



Points of P (places or classes of equivalence of individuals) are the relations between points of P^* and relations between points of P are the points of P^* (the institutions).

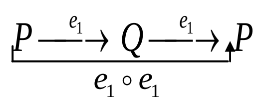
This duality is very interesting. *The places can be conceived as points of a network as well as the relations between points of a network.* If we conceive social structure as a network where different entities, material and informational, flows, then the description of its structure by means of the concepts here defined allows us to consider the individuals or their classes of equivalence either as points through which something flows or as channels by which this circulation takes place.

The study of the network of places P or of its dual network P^* , nevertheless, can not be limited to the analysis of direct relations between the points. Indirect relations, concatenation of direct relations, may have a great structural signification in the network.

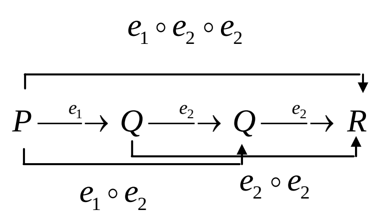
7.- The chains of links in the networks of places

Following the usage established by Lorrain and White (1971), we will call *morphisms* the relations e_k between the points of a network of places, as well as *any chain of such relations*. These morphisms represent the direct or indirect relations between places.

Among the morphisms, we can distinguish between the generator morphisms, e_k, that are the elements of E, and the composed morphisms, chains of generator morphisms. Then, in our example, we will have a composed morphism $e_1 \circ e_2$ from P to P.



There are also morphisms $e_1 \circ e_1$ from P to Q, from Q to P and from Q to Q. Here we have other examples of composed morphisms:

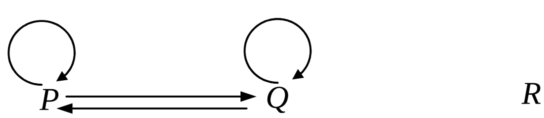


The graph of a generator morphism e_k is the set of couples (X,Y), where X and Y are places belonging to e_k (containing e_k). For example, if

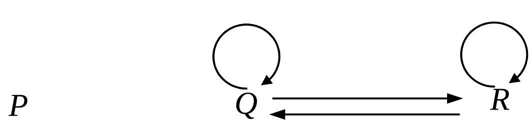
$A_1 = \{P, Q\}$ is the set of places belonging to e_1 ,

The graph of e_1 is simply the cartesian product $A_1 \times A_1$, that is to say, the set of all the couples of elements of A_1

Graph of e_1



Equally, the graph of e_2 will be simply $A_2 \times A_2$

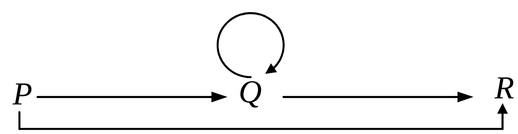


Graph of e_2

The composed morphism $e_1 \circ e_2$ is defined simply because there exist individuals who belong simultaneously to both institutions, that is to say, because there exists a place (in this case, Q) containing both institutions. That is equivalent to say that the intersection $A_1 \cap A_2$ is non empty. It is the same for any other composed morphism $e_i \circ e_k$.

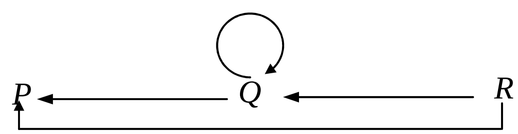
Which is the graph of the composed morphism $e_1 \circ e_2$? It is the set of all couples (X,Y), where X is a place belonging to e_1 (containing the element e_1) and Y a place belonging to e_2 (containing e_2). In other words, the graph of the composed morphism $e_1 \circ e_2$ is the cartesian product $A_1 \times A_2$.

graph of $e_1 \circ e_2$



equally, the graph of $e_2 \circ e_1$ is $A_2 \times A_1$:

graph of $e_2 \circ e_1$



We observe a very interesting thing: while the composed morphism $e_k \circ e_1 \circ e_m$ is defined (that is to say, if there is such a chain of relations between the points of the network of places P and, what is the same, that intersections $A_k \times A_l$ and $A_l \times A_m$ are non empty), then the graph of the composed morphism $e_k \circ e_1 \circ e_m$ is simply equal to $A_k \times A_m$. The same way, if the composed morphism $e_k \circ e_1 \circ e_m \circ e_n$ is defined, then its graph is simply $A_k \times A_n$, and so on.

Because as we have seen the morphism $e_k \circ e_k$ as well as all the defined morphisms of the type $e_k \circ \ldots \circ e_k$ have all the same graph that e_k, we can consider all these morphisms as corresponding to the same social relation and, consequently, identify all those composed morphisms to e_k:

(1) $e_k \circ \ldots \circ e_k = e_k \circ e_k = e_k$.

Similarly, all the defined morphisms $e_k \circ \ldots \circ e_1$ have the same graph as $e_k \circ e_1$ and we will consider them as representing the same social relation and are thus be identified to $e_k \circ e_k$:

(2) $e_k \circ \ \ldots \ \circ e_1 = e_k \circ e_1$.

The equations (1) and (2) express *the law of the first and the last letters* (LORRAIN: 1975).

Let's focus now on the law of morphisms composition in the case where there are only two generating morphisms, as in our example. To simplify the notation we will equate

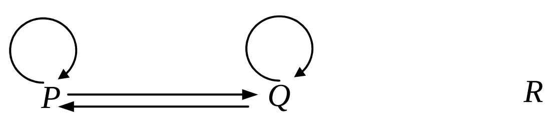
$e_1 = a$ and $e_2 = b$

Following the law of the first and the last letters, every composed morphism has to be equal to one of the four morphisms *a*, *b*, $a \circ b$ or $b \circ a$. The composition table of the morphisms is the following one:

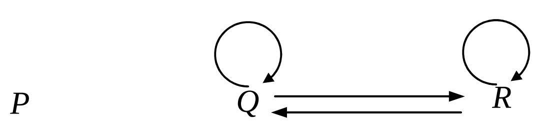
	a	b	$a \circ b$	$b \circ a$
a	a	$a \circ b$	$a \circ b$	a
b	$b \circ a$	b	b	$b \circ a$
$a \circ b$	a	$a \circ b$	$a \circ b$	a
$b \circ a$	$b \circ a$	b	b	$b \circ a$

In our very simple example, graphs of the four morphisms are the following:

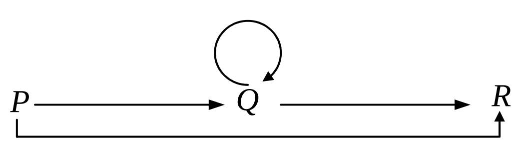
Graph of *a*:



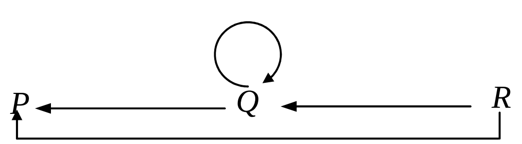
Graph of *b*:



Graph of $a \circ b$:



Graph of $b \circ a$:



We will note that, while generating morphisms *a* and *b* have symmetrical graphs, composed morphisms have non symmetrical graphs. *We find again here a very important aspect of social structures: the non symmetrical social relations*.

The set of morphisms (a, b, $a \circ b$, $b \circ a$), together with their composition table constitutes what is called a *semigroup.* This semigroup *S* has been generated by the set of institutions ($e_l = a$ and $e_2 = b$) and each element of *S* is idempotent ($e_k \circ e_k = e_k$) and S is a *rectangular band* (CLIFFORD & PRESTON, 1961). In this particular case, it should be called a *square band*, a particular case of a rectangular band.

This semigroup *S* has notable properties. One of them is that a and b have isomorphic places in the composition law: if we substitute everywhere in the composition table a by b and b by a, we got the same composition table, with a permutation of lines 1 and 2 and of lines 3 and 4, as well as columns 1 and 2 and columns 3 and 4. All this changes nothing to the behavior of morphisms. In a technical language, the permutation

$$\begin{Bmatrix} a & b & a \circ b & b \circ a \\ b & a & b \circ a & a \circ b \end{Bmatrix}$$

is an automorphism of the semigroup *S.*

Then, nothing *in the composition table* allows us to distinguish between social relations a and b of our network of places. On the contrary, of course, graph of *a* in the network of places is different from graph of *b*, because of the different set of places related by *a* and the particular set of places that *b* relates, these two sets of places may contain different numbers of places and their intersection can be more or less important.

There is other interesting – and strange – property of this semigroup. It is possible to take as generating morphisms $a \circ b$ and $b \circ a$ and find, by means of the compositions laws, *a* and *b* as composed morphisms:

(3) $a \circ b \circ b \circ a = a$ and $b \circ a \circ a \circ b = b$

This last fact is, from a sociological standpoint, of a great signification: we can consider the generating relations a and b, that is to say, the institutional memberships defining places, as derived from indirect relations, the composed morphisms. Intra institutional relations may be viewed as derived from inter institutional relations. We can, then, *conceive places as defined by their mutual relations*, which is the minimal condition of any structural thought in social sciences. Also, in what concerns social actors subjectivity, this last fact allow us to consider *consciousness of membership as a fact derived from unconscious indirect social relations*. Subjectivity is no longer a prior condition for social structure.

Equations (3) imply that, if we replace in the composition table:

a by $a \circ b$, $a \circ b$ by a, b by $b \circ a$ and $b \circ a$ by b,

we got the same composition table, with permuted lines and columns. The permutation:

$$\begin{Bmatrix} a & b & a \circ b & b \circ a \\ a \circ b & b \circ a & a & b \end{Bmatrix}$$

is also an automorphism of semigroup *S*. That is to say, *a* and $a \circ b$ are in isomorphic positions in *S*, as well as *b* with $b \circ a$.

We know also that the set {*a*, $a \circ b$ } constitutes a sub semigroup of *S*, as well as does the set {*b*, $b \circ a$ }, both sub semigroups being rectangular bands.

Also, the permutation:

$$\begin{Bmatrix} a & b & a \circ b & b \circ a \\ b \circ a & a \circ b & b & a \end{Bmatrix}$$

is another automorphism of *S* and we can say that *a* and $b \circ a$ are in isomorphic positions in *S*, as well as *b* and $a \circ b$. The set $\{a,\ b \circ a\}$ constitutes a sub semigroup of *S*, in the same way as the set $\{b,\ a \circ b\}$ being both sub semigroups and rectangular bands.

8.- Places and Networks of Places in the Spanish Power Elite during the Transition Period (1975-1981)

For illustration purposes, we include here a brief analysis of networks of places from a real data set .

This data set is a result of a research that followed the analysis of the Spanish Power Elite in the dictatorship period (1939- 1975), Baena M. and Pizarro N. [1985], partially analyzed in Baena, Garrido and Pizarro [1984]. It covers the Spanish power elite individuals and institutions existing during the Transition to democracy period, 1975- 1982.

It consists of 11103 individuals and 1474 membership groups: Directorates of big and medium sized firms, parlamentary groups of the existing political parties, sets of high level positions in the Public Administration and the Executive and civil service elite Corps. This dual graph is a complete graph, not a sample: it includes the totality of the individuals fulfilling the conditions previously defined for elite membership.

After using a SAS/IML program written by Sebastien Delarre (Lille 3 University), we have finally developed a very fast program called TextToPajek[1] (Perez & Pizarro, UCM, 2004) written in C++, used here for the first analysis of the already mentionned data set. This program generates places from either text files or PAJEK dual file format and produces the corresponding network of places also in PAJEK .net format[2].

Results:

We found a total of 3582 places. Of these 1336 places are occupied by more than 1 person, 2562 include more than 1 group and 235 include more than 1 person and more than 1 group. We expose the results of the analysis of the whole network of places, where points are places and links institutions, using currents tools for SNA.

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[1] Available for download at www.ucm.es/pecar/

[2] These formats are documented in Pajek's Manual, avalaible at http://vlado.fmf.uni-lj.si/pub/networks/pajek/

The whole network of places found has:

Number of vertices (n): 3852

	Arcs	Edges
Number of lines with value=1	0	79499
Number of lines with value#1	0	9417
Total number of lines	0	88916
Number of loops	0	0
Number of multiple lines	0	0

Density1 [loops allowed] = 0.0119850
Density2 [no loops allowed] = 0.0119881

This network of places has a giant connected component of size 3647 (94,678%), one size 4 component, five components of size 3, 2 components of size 3 and 186 isolated places.
And this giant connected component includes one giant bi-component, of size 3414 and 17 small bi-components, of sizes lower than 5.

The network of places has a diameter of 9 and the distribution of distances between all the points is the following:

Average distance among reachable pairs: 3.34095
The most distant vertices: P97(1,7) (97) and P263(1,5) (263). Distance is 9.

Viewing Vector --- 2. Distribution of Distances in N1 (9)	
1.	177832
2.	2169818
3.	5565778
4.	3973938
5.	1172838
6.	209906
7.	24906
8.	1904
9.	96

Let's not forget that what we are analyzing here is a networks of places, as previously defined. And that every place corresponds to an equivalence class in the set of individuals. That is to say that the complete set of places defines a partition on the set of individuals. While the network defined on the complete sets of places is connected, sparse, locally dense and with an average distance between pairs of 3.34, the corresponding equivalence classes are, obviously, not connected at all by their intersections. Structurally equivalent idividuals are connected only by the intersections of their corresponding places.

This network of places is the a fairly dense and connected network.

The dual network, a network of institutions connected by places has the following attributes:

Number of vertices (n): 1484

	Arcs	Edges
Total number of lines	0	12586
Number of loops	0	0
Number of multiple lines	0	3909

Density1 [loops allowed] = 0.0114301
Density2 [no loops allowed] = 0.0114378

Network density is lower than in the network of places: 0.01143 against 0.0119881.

The distribution of distances is the following in this dual network:

Average distance among reachable pairs: 3.38689
The most distant vertices: 2782E (171) and 0441A (401). Distance is 8.

Distribution of Distances in N5 (8)

1.	17354
2.	224828
3.	709940
4.	524398
5.	149028
6.	22352
7.	1960
8.	108

Then, the diameter of the dual graph is lower than in the network of places: 8 against 9.

It has also a giant component of size 1285,   86.59% of the network size.

This dual network is also a fairly dense and connected network.

The study of these networks is just starting and the analysis of composition of morfisms is to be done. Lets mention only that second order morfisms found are 17354 !.

We wanted to present here enough evidence to demostrate the interest of the networks of places for sociological research.

9.- Conclusion

LORRAIN and WHITE (1971) article has been widely read and quoted . The idea of structural equivalence as a tool for reducing networks of individuals by the way of substituting individuals by classes of structurally equivalent individuals has been accepted, with the only restriction of its applicability to real situations – where it is difficult to find structurally equivalent individuals – and also, because the amount of calculus that is necessary to find them. It has been a very interesting idea, a fascinating concept and a complex mathematical construction.

Nevertheless, the other main idea that LORRAIN and WHITE develop in their article has been lost. And it was, at least from my point of view, the most important: *compound relations can have determinant structural consequences in social networks*. It is a fascinating idea, because compound social relations do not appear in the consciousness of the individuals whose direct relations are determined and structured by them.

In the later article, the social effect of a compound, indirect relation, namely $n \circ n^{-1}$ (where n^{-1} is the reciprocal of relation *n* that a individual *a* has with his enemy *b*), was shown to explain the actual divisions of a concrete group of individuals: they came together in empirically observed subgroups, that are components of the graph of this compound and indirect relation (whose interpretation is: $a\left(n \circ n^{-1}\right)b$ means that *a* and *b* are linked by the existence of, at least, a common enemy *x*) with an individual outside the subgroup. This is the logic of coalitions, that has a very clear sociological interpretation. But here it appears out of subject's intents, as it is derived from the actual grouping of subjects.

My hypothesis concerning why such an important intuition has not been taken into account in contemporary research on social networks is very simple. The social effect of indirect, compound relations is in contradiction with the ideological postulates of voluntaristic individualism, with the explanation of social action as a set of interacting individual actions, each one of them driven by individual's ideas, values and motifs. Indirect, compound social relations are neither conscious nor voluntary and, consequently, they have no place in the current academic interpretation of structures and processes.

LORRAIN and WHITE's article has had part of the responsibility of this state of affairs: it deals with networks of *individuals* and, doing so, focuses the attention on the most traditional perspective in contemporary social sciences: units of analysis are always individuals, primary relations are, also always, relations between individuals. The composition of relations was interpreted by most of the readers as a complicated procedure to get the initial network reduced, to build classes of equivalent individuals.

The concepts of place and of network of places here developed starts from a different methodological standpoint: social relations should not be defined as relations between individuals because those relations are not full fledged *social relations*: they cannot be regular over long period of time, being defined without any discussion of the identity of individuals or, equivalently, without taking into account the social processes that are responsible for those differentiated identities. Places, on the contrary, are independent of the identities of the individuals who occupy them, either simultaneously or not.

Networks of places are networks of a kind of social relations that are not necessarily facts of consciousness, as are inter-individual relations. Places are not social subjects or actors but structurally defined social entities which have not consciousness, ideas, values, norms nor motifs. That is why they are, initially, a concept difficult to understand and to use as a tool for social research.

Nevertheless, networks of places offer a direct and simple way to capture social facts: in a network of places, individuals or – better, classes of structurally equivalent individuals- are *identified* by the place they occupy. Individual's identity is then strictly positional. And places can be defined either by "primary" institutions – the membership groups in the raw data – or by the relations between places. But the structure of networks of places is, as it has been shown above, such that *"primary" institutions can also be conceived as the result of composition of relations*. This last property implies that not only individual's identities are relationally defined, but also that "institutions" or primary membership groups can be defined afterwards, and explained, by the mean of the composition of compound morphisms, considered then as primary data.

Simmel's concept of *social circles* as a source of individual's identity and also of the inter individual relationships has been interpreted merely as rude and pre-technical definition of the so called "ego centered network", losing then the main founding intuition of the German sociologist. For Simmel, social circles are also defined by their mutual relationships: their intersection is something more (structurally) significant than a mere subset of individuals. And this is so because, for Simmel as for us, *individual identity is not given as a pre social, pre relational, fact*. Then, individuals have as identity their social relations and, reciprocally, social relations can be viewed as particular subsets of individuals. In sociological theory no concept can exist outside – prior to – its sociological definition, unless we accept that social sciences are no more than applied psychology, a new conception of moral or a branch of biology.

But the concept of place is defined as a subset of the set of institutions, and not as the intersection of two or more social groups. The intersection of social groups are subsets of individuals. The duality of individuals and groups that Breiger underlines (Breiger: 1974), means that we can look at social reality taking into account two networks, the network of individuals and the network of groups, that are dual because relations between groups are individuals and relations between individuals are groups. Networks of places, as above defined, have as points the set of places, and as relations between points, the intersection of places, two dual networks are no more needed to capture social structure, because no network of individual relations is needed.

Neither Simmel's ideas, nor the conception of the duality of individuals and groups (Breiger:1974) that constitutes the current formalization of these ideas, are equivalent to the concept of places and networks of places. Networks of places are social networks, but social networks socially defined: the "a priori" definition of individual's identities and of membership group's identifications becomes a starting point of the analysis that is transformed by the analysis itself, as it explains its owns origins as by-products of inner structure.

Furthermore, using identified (distinct) individuals as units of analysis implies that the time intervals inside which we can observe whether regularities appear are biologically delimited. Networks of individuals have a short life and thus can not be used as an analytical tool for assessing whether relational patterns are or not invariant over time period longer than average human life. As soon as a social scientist works, as we did, with data sets covering five decades and thousands of individuals belonging to institutionalized membership groups that may also modify their formal identities over the period of time under exam, the critique of the concepts used in social network analysis is not simply a theoretical question. It becomes an empirical need, as it is the construction of a new conception of a social network allowing research on available data on fundamental aspects of social reality. Obviously, this need is not felt if research is limited to a mere mathematical modeling of a social reality that is little more than an allusion to an external set of ill defined facts.

Finally, a short comment on the purpose of research in social network analysis in particular, and in structural sociology in general. Let's suppose that we will be able to discover interesting social structures, particular shapes or forms in social networks that bring a new light on our experience of society. We will then say: so it is...

But scientific endeavor has always had other aims. We have need to know also why these forms appear, and not only what is their shape. Metaphorically, we wait for the transition from statics to dynamics. But this transition, in physics, required first the development of kinematics, the description of the changes of position in time of invariant objects.

The purpose of the construction of networks of places is to seek a tool for describing changes in time of invariant patterns. In some way, the present work pretends to be a step toward the construction of a "kinematics" for the social sciences. But in no way it tries to be an explanation. In our field of research, dynamics will be something similar to the socio-genesis of social structures.

The emergence of regularities – patterns of networks of places invariant in given time intervals – could be the starting point for a research in the dynamics of social structures. A few ideas could be worth to be explored:

1.- The regularities appearing in structural analysis could be considered as the outcomes of processes of regulation.

2.- What is regulated should be the social production of social entities, understanding as such social subjects and positions, their relations, but also all other social products; objects that are not produced by nature are as social as humans beings are supposed to be. Then, categories and concepts have to be thought as socially produced.

3.- That processes of regulation should be also socially produced, at least if we want to remain in the attempt of providing a social explanation of social facts.

These three points were the basis of previous attempts of sketching a theory of social reproduction [Pizarro: 1972, 1999], that is probably still far from maturity.

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