# International Sociology

http://iss.sagepub.com


## Structural Identity and Equivalence of Individuals in Social Networks: Beyond Duality


Narciso Pizarro



*International Sociology* 2007; 22; 767
DOI: 10.1177/0268580907082260

# Structural Identity and Equivalence of Individuals in Social Networks

## *Beyond Duality*

Narciso Pizarro
*Universidad Complutense de Madrid*

***abstract:*** There are two problems in structural sociology that have attracted the attention of researchers: first, the creation of individual identity by means of the social networks in which individuals are embedded; second, that of structural equivalence, i.e. the problem of non-identification of individuals by means of those social networks. The aim of this article is to bring both problems together under a common framework from which both identity and equivalence of individuals in social networks will simultaneously emerge. Or in other words, the study attempts to show that structural determinations of identity and equivalence are just the same. In order to achieve this, it is necessary to go beyond the duality approach insofar as duality represents social structure *either* as a network of interconnected social circles *or* as a network of individuals who are linked by one or some given relations. Accordingly, the concept of place and place analysis is used.

***keywords:*** duality ✦ network identity of individuals ✦ networks of places ✦ places ✦ structural equivalence ✦ two-mode networks

## Introduction

The recent discussions of Simmel's contribution to structural social sciences (Breiger, 1990) have not yet completely reached the mathematical modelling of networks. In Simmel's terms, individual identities are considered a property of the network of relations between membership groups, called social circles. This follows the basic idea of all structural analysis that entities should be defined as a result of the relational systems and not independently from them. However, there is then a gap between the actual

International Sociology ✦ November 2007 ✦ Vol. 22(6): 767–792

SAGE (Los Angeles, London, New Delhi and Singapore)
DOI: 10.1177/0268580907082260

level of theoretical understanding of structural concepts and the basic mathematical modelling. This gap appears in social network analysis when social networks are defined as a set of individuals as nodes and a set of inter-individual relations defined as contingent phenomena concerning individual nodes while looking for social structure by these means.

There are two problems in structural sociology that have attracted the attention of researchers. First, the relational definition (and then, creation) of individuals' identity by means of the social networks in which they are embedded. Second, that of finding in a network individuals that are structurally equivalent, that is to say, individuals whose relations to all others are identical and, consequently, cannot be relationally distinguished, i.e. the problem of non-identification of individuals by means of those social networks. These two problems have been analysed separately, both in their theoretical approach and in their methodological treatment. The first, coming from the Simmelian tradition, has been discussed using sets theory and combinatorial mathematics (Breiger, 1990, 2000). The second, coming from sociometry, has been mainly algebraic in its methodological proposals.

The search for structurally equivalent individuals is an attempt to reduce the size and complexity of social networks, building networks in which the nodes are classes of equivalent individuals and the arcs or edges linking these classes are the ones that existed in the original network.

Our aim is to unify both problems under a common framework from which both identity and equivalence of individuals in social networks will simultaneously emerge. Or in other words, we attempt to show that structural determinations of identity and equivalence are just the same.

In order to achieve our purpose, we necessarily need to go beyond the duality approach insofar as duality represents social structure *either* as a network of interconnected social circles *or* as a network of individuals who are linked by one or some given relations. The concept of place and place analysis allows us to build social networks that take into account simultaneously individuals and social circles.

## What Do We Mean by Individuals?

When we say: let's consider two individuals, A and B. How do we know that they are different from each other? The answer to this question seems so obvious that we never discuss it. Nevertheless, individual identity is not a simple matter.

The attributes used to differentiate individuals are mainly social. The individual's name is a social institution, regulated by custom, and by civil law in contemporary states. Names are inherited, and so related to parenthood systems, the primordial social institution. Names mean mainly 'Individual A is a member of a family, X'. She or he is a daughter/son of a particular couple of other individuals.

Other attributes used to identify individuals are also social institutions: professions can be viewed as the names socially given to individuals who belong to specific social groups. To say 'A is a lawyer' means also 'A is member of the Bar'. Most of the characteristics of individuals, including professions, with the main exception of physical attributes of the body, are socially constructed categories, related to social groups with a certain level of institutionalization.

Accordingly, it is possible to *define individual identity as a set of membership relations*. This definition of identity is, as we see later, an operational concept that can be used as a powerful tool in structural social sciences. As Breiger (1990) shows, Simmel's (1927) ideas on the intersection of social circles can be understood as a logic of identity deriving from multiple memberships.

## What Do We Mean by Social Relations?

The expression *social relation* is at least as seemingly obvious as the meaning of the term *individual*: social relations are relations between individuals. Social sciences take them as facts because they seem to appear like facts in individual consciousness.

Social research constructs data sets of social relations, asking individuals about their social relations, and noting their answers in a more or less careful manner. Registered answers are very heterogeneous: some of them refer to emotions, other to values and attitudes, both merely psychological entities. But some species of answers refer also to objective social processes, in a way that, because of their psychological nature, it is not always easy to relate to the social facts we are looking for.

Since Nadel (1956), we know that not all the individual's interactions have to be studied by social sciences. Some interactions are merely random events, never reproduced, that cannot or should not be confused with social relations: the expression social relation should be used only for patterns of interaction that are *independent from the particular individuals involved* in them. Those patterns should also regularly exist *in time, over periods longer than an individual's human life*.

Some psychological entities noted in empirical research respect the preceding criteria, but many don't. It is not easy, from a strictly empirical standpoint, to distinguish one from another in what concerns their individual character or their properties in time.

But not all of them are *only* psychological entities: in the individual's answers, we get subjective references to social facts that can be observed in the outside world, beyond the individual's consciousness.

Traces of many social relations are kept in *archives, where their inscription is institutionally produced*: inscription is a social process, which depends upon the relation's compliance with rules and norms. And it implies the acknowledgement of the relation by a social group, in such a way that it is no longer an intersubjective matter, but a socially regulated social relation: a marriage is more than an inter-individual affair as the intersubjective relation becomes properly published, recognized and deserving an inscription in the collective memory.

## Membership Relations and their Sociological Meaning

The individual's membership to a particular social group is a very general kind of social relation, embedded in our ways of talking about ourselves and about our relations with others. What has been said before about individual identity is a very important part of the sociological meaning of membership relations but this is not the whole story: there is another aspect.

In addition to its contribution to the definition of identity, *membership determines also the nature and contents of inter-individual interaction*. It is because subject X belongs to the group of physicians that he can have a particular relation with subject Y, involving the possibility for X of modifying Y's behaviour (eat or drink a particular substance), as Y recognizes X's membership to the group of doctors and himself as needing a doctor's care.

The concept of role is not an adequate instrument to capture this very important aspect of social structure: the social existence of a doctor has more contents than those captured by the identification of an individual in a doctor's role. In the same way, relations between corporation board members cannot be defined and understood from their mutual interaction, forgetting the determinant character of their common belonging to the same board.

Common memberships explain the content of relations better than data proceeding from subjective descriptions, and it is more so when the relations under study are, as they should be in social sciences, de-individualized and long-lasting types of interaction. Only for the types of relations that Simmel considers as merely inter-subjective do membership's determinations lose their explanatory power.

## Overlapping Social Circles and the Problem of Duality

Breiger (1990, 2000) deals with the problem of individualization of human beings in social networks starting from Simmel's ideas on that question. Following Simmel, if a given person is *the only one* that exists in the intersection of a *particular subset* of social circles, then this person is completely individualized. This person's identity is consequently given by the intersection of the social circles to which this person belongs.

The social circles we are talking about here are strong *socially and organizationally produced* entities: membership in these social circles is never a product of the member's free will alone, nor the result of random allocation processes. *Membership, on the contrary, is submitted to social rules and institutionalized social processes.*

Social structure has a fundamental feature. It can be seen as a set of social circles linked by common members, or as a set of individuals linked by their common membership to the same circles. Breiger (1974) shows that the networks of circles connected by individuals and those of individuals connected by common circles are both derived from the incidence matrix, a $p$ by $q$ matrix $\boldsymbol{A}$, where $p$ is the number of individuals and $q$ the number of social circles, which expresses the membership relation of individuals to groups.[1]

Let $\boldsymbol{C} = \{X1, X2 ....... Xn\}$ be a set of individuals and $\boldsymbol{G} = \{G1, G2 ....... Gm\}$ a set of social circles, constituted by individuals belonging to $\boldsymbol{C}$.

We will call the *incidence matrix* a rectangular and binary matrix $\boldsymbol{A}$.

| | | | | |
|---|---|---|---|---|
| 1 | 1… | | 1 | |
| 0 | 0 | | 0 | |
| $\boldsymbol{A}$ = | 1 | 1 | | 1 |
| 0 | 1 | | 0 | |

that represents the membership of individuals belonging to set $\boldsymbol{C}$ to social circles of set $\boldsymbol{G}$. The intersection of row $i$ with column $j$ is zero if the individual $X_i$ does not belong to social circle $G_j$ and is equal to 1 if it belongs to it.

If we consider that common membership of two or more individuals to the same social circles defines a social relation between the individuals, we can then conceive the incidence matrix $\boldsymbol{A}$ as representing a relations network in which there are so many relations as social circles to which it is possible to belong.

Let incidence matrix $\boldsymbol{A}$ be the following:

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |

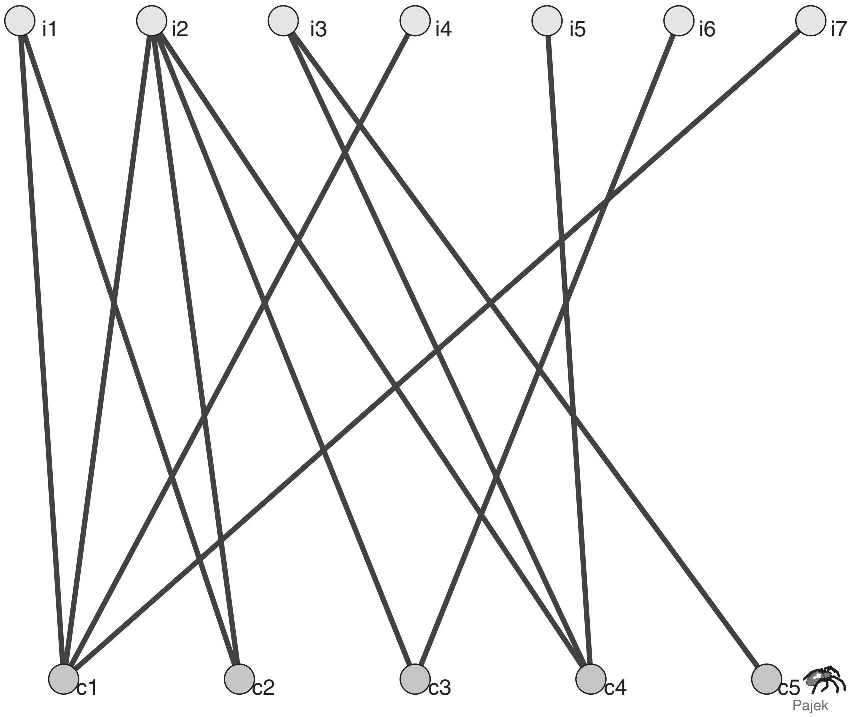



***Figure 1*** *Membership Relation of Seven Individuals to Five Social Circles*
*Note:* All the networks in this article are drawn using Pajek.

It has seven individuals (seven rows) and five social circles (five columns), and membership relations of the seven individuals to the five social circles can be represented by the two-mode graph shown in Figure 1.

Considering that if two individuals belong to same social circle, they are in a relation coming from the social circle they both belong, we can represent relations between individuals by the mean of a square matrix ***I*** in which the seven rows and columns represent individuals, and their intersection, their relation:

| | 1 2 3 4 5 6 7 |
|---|---|
| Individual 1. | 0 2 0 1 0 0 1 |
| Individual 2. | 2 0 1 1 1 1 1 |
| Individual 3. | 0 1 0 0 1 0 0 |
| Individual 4. | 1 1 0 0 0 0 1 |
| Individual 5. | 0 1 1 0 0 0 0 |
| Individual 6. | 0 1 0 0 0 0 0 |
| Individual 7. | 1 1 0 1 0 0 0 |

We see that it is not a binary matrix, since individuals 1 and 2 are simultaneously in the same two social circles.

The same incidence matrix $\boldsymbol{A}$ gives a network of relations between social circles, in which the edges are the individuals that two social circles have in common, and the nodes are the social circles. The following adjacency matrix expresses the number of individuals in common for each pair of social circles. Let this matrix be matrix $\boldsymbol{S}$:

| | | 1 2 3 4 5 |
|---|---|---|
| SocCir.1 | 1. | 0 2 1 1 0 |
| SocCir.2 | 2. | 2 0 1 1 0 |
| SocCir.3 | 3. | 1 1 0 1 0 |
| SocCir.4 | 4. | 1 1 1 0 1 |
| SocCir.5 | 5. | 0 0 0 1 0 |

The three matrices, $\boldsymbol{A}$, $\boldsymbol{I}$ and $\boldsymbol{S}$, are in a relation that we call duality. In simple terms, duality is the relation between a network of individuals linked by common social circles, and a network of social circles is a set of social circles linked by common individuals, when the two networks are derived from the same affiliation matrix.

Let $\boldsymbol{A}^T$ be the transposed matrix of $\boldsymbol{A}$ (i.e. the matrix obtained transforming rows into columns and columns into rows). It is easy to prove (Breiger, 1974) that:

$$\boldsymbol{S} = \boldsymbol{A} \times \boldsymbol{A}^T$$
$$\boldsymbol{I} = \boldsymbol{A}^T \times \boldsymbol{A}$$

## The Concept of Structural Equivalence of Individuals

In a different context, looking for an explanation of the formation of cliques in a group of individuals, Lorrain and White (1971) developed the concept of structural equivalence of individuals in social networks. In this approach, directed graphs are analysed in order to look for individuals whose relations with the remaining individuals in the network are identical.

When all the structurally equivalent individuals have been grouped in classes of equivalence, we obtain a reduced network, with fewer nodes, but conserving the same structure: it can be conceived as a skeleton network.

The concept of structural equivalence is, in the original definition, very strict, and the algorithms developed for the search of structurally equivalent nodes in the network are very demanding in computational terms. For that reason, less strict approaches – such as block modelling – have been developed since the original article, ‘Social Structure from Multiple

Networks: I. Blockmodels of Roles and Positions' (White et al., 1976). Nevertheless, even nowadays block modelling algorithms are only efficient for networks of fewer than 1000 nodes, as the amount of calculations varies in a hyper-quadratic way with respect to the number of nodes.

Block modelling has extended to be applied to two-mode networks. However, this extension does not take into account the specificity of the membership relations from which these graphs emerge and, in practice, it functions as a generalization of block modelling as initially thought for networks of individuals or circles (see Doreian et al., 2004); but it does not work for essentially dual entities, which unify both representations of structure.

Besides, block modelling does not search for structurally equivalent individuals, but for roles that derive from inter-individual relations.

## Places, Networks of Places and Duality

The preceding considerations came, of course, after the need for a new concept became evident, one more adequate to capture the de-individualized and time-regular character of social relations, and as it became evident how to define it. Dealing with data about multiple memberships of thousands of individuals over decades or long periods of time, it is clear that the following definition is, for the time being, a convenient research tool.

If we have a finite set of individuals:

$$I = \{i_1, i_2, i_3, \dots i_p\}$$

each belonging to one or more *socially* defined sets of individuals, here called *social circles* to stress with the word the *socially built* character of the set and noted

$$E = \{e_1, e_2, e_3, \dots e_q\}$$

We define a *place* $P_i$ of an individual $i \in \boldsymbol{I}$ as a subset of $E$ such that *at least one* of the individual members of $I$ belongs to every one and only to the social circles included in the subset $P_i$. That is to say, for $i \in I$

$$P_i = \{e_j \in E : i_i \in e_j\}$$

If two individuals $i, j \in I$ have the same subsets of $E$, their place is the same. So defined, *places are subsets of E*, independent from individuals.

Also, social structure can be defined as a network of places, using the following straightforward definition of a relation between two places:

Definition 2: Two places $\boldsymbol{P}_i$ and $\boldsymbol{P}_j$ are in a relation $\boldsymbol{R}$ if $\boldsymbol{P}_i \cap \boldsymbol{P}_j \neq \emptyset$

The set $\boldsymbol{P}$ of all the places defined in $\boldsymbol{E}$ and the set $\boldsymbol{R}$ of their relations constitutes a *network of places.*

Given the preceding definitions, networks of places are *reductions of the initial dual networks that conserve the duality.*

## Structural Equivalence and Networks of Places

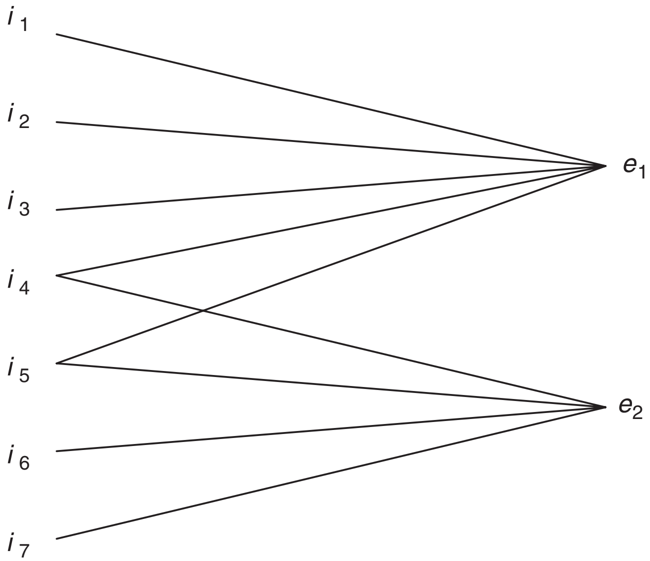
In a network of individuals, we can identify sets of structurally equivalent individuals (Lorrain and White, 1971). We can look at structurally equivalent individuals in a network of membership relations or dual network. We have a set of individuals:

$$I = \{i_1, i_2, i_3 \ldots i_p\}$$

and a set of social circles

$$E = \{e_1, e_2, e_3, \ldots e_q\}$$

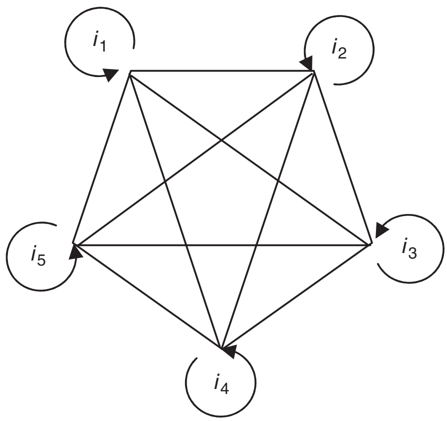
each individual is a member of a certain subset of social circles. For example, if the cardinal of set $I$'s is seven, where $E$ had two elements, we could have the following membership relations:



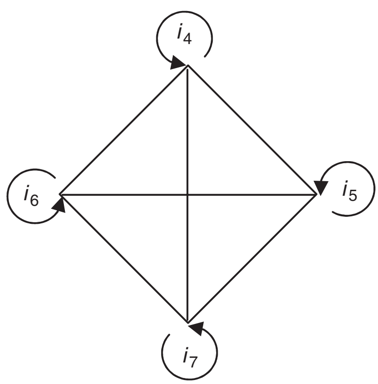
We consider an individual $i_r$ to be linked to an individual $i_s$ through a link $e_k$ if both individuals are members of $e_k$:

$$i_r \xrightarrow{e_k} i_s \text{ if } i_r, i_s \in e_k$$

In our example, we will have the following links of type $e_1$ between individuals:




and links of type $e_2$ between individuals:




In a network, *two individuals are structurally equivalent if they are both members of, precisely, the same social circles*. In our example, individuals $i_{1,}$ $i_2$ and $i_3$ are all members of the same social circle $e_1$ and of no other; consequently:

$i_{1,}$ $i_2$ and $i_3$ are structurally equivalent.

Individuals $i_4$ and $i_5$ are both members of social circle $e_1$ and $e_2$ and to no other, and so:

$i_4$ and $i_5$ are structurally equivalent.

Similarly, $i_6$ and $i_7$ are members of $e_2$ and they do not belong to any other social circle, thus:

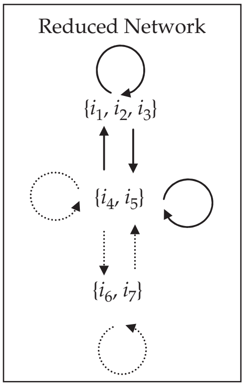
$i_6$ and $i_7$ are structurally equivalent.

Two individuals are structurally equivalent if the occupy the same *place*, in the sense previously defined. *Each class of equivalent individuals corresponds to a place*, that is to say, to the set of social circles that defines the place itself. In our example, we have three classes of equivalence of individuals corresponding to the following places:

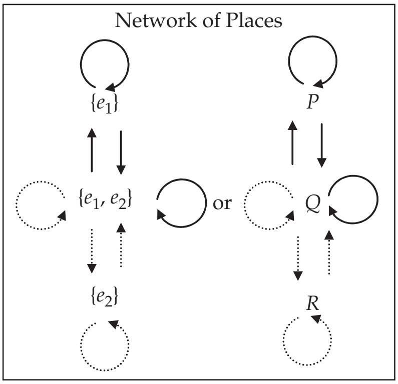
| Equivalence classes | Corresponding places |
|---|---|
| $\{i_1, i_2, i_3\}$ | $\{e_1\}$ |
| $\{i_4, i_5\}$ | $\{e_1, e_2\}$ |
| $\{i_6, i_7\}$ | $\{e_2\}$ |

Each class of equivalence of an individual corresponds to a unique place and, reciprocally, each place defines a unique equivalence class of individuals. The expressions *equivalence class of individuals* and *place* are practically synonymous.

If structurally equivalent individuals are identified, a reduced network is obtained, where points are equivalence classes, that is to say places. In our example, the reduced network has the following appearance, where links $e_1$ are represented by continuous arcs and those of $e_2$ type, by dotted arcs.




or the equivalent



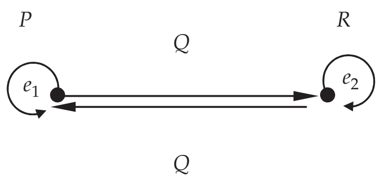

A place $X$ is linked to a place $Y$ by a link $e_k$ if individuals belonging to $X$ and to $Y$ all belong to social circle $e_k$ – that is to say, if social circle $e_k$ belongs simultaneously to the set of social circles of place $X$ and to that of place $Y$:

$$X \xrightarrow{e_k} Y \text{ if } X, Y \supseteq e_k$$

We saw how the network of individuals linked by social circles can be reduced to a *network P of places linked by social circles*. It is possible, also, to define a dual network: a *network P* of social circles linked by places*. In this dual network a social circle $e_k$ is linked to a social circle $e_l$ by a link $P$ if these social circles both belong to the place $P$:

$$e_k \xrightarrow{p_r} e_l \text{ if } e_k, e_l \in P$$

In our example, the dual network $P^*$ is the following:




Points of $P$ (places or classes of equivalence of individuals) are the relations between points of $P^*$ and relations between points of $P$ are the points of $P^*$ (the social circles).

This duality is very interesting. *The places can be conceived as points of a network as well as the relations between points of a network.* If we conceive social structure as a network where different entities, material and informational, flow, then the description of its structure by means of the concepts here defined allows us to consider the individuals or their classes of equivalence either as points through which something flows or as channels by which this circulation takes place.

The study of the network of places $P$ or of its dual network $P^*$ may not, nevertheless, be limited to the analysis of direct relations between the points. Indirect relations, concatenation of direct relations, may have a great structural signification in the network.[2]

## Using Networks of Places with One-Mode Networks

Networks of places are not only useful for the analysis of dual networks. In one-mode networks, it is possible to identify cliques and use them as the social circles already mentioned to build a dual network.

Freeman (1996) uses the same procedure to build Galois lattices from one-mode networks. That is to say:

1. First, he extracts from the one-mode network all the cliques, using the standard clique definition and the usual algorithms and programs to identify them.

2. He uses the initial set of cliques to build the corresponding Galois lattice.
3. He uses the Galois lattice analysis to identify social groups.

We follow here a similar procedure, using sets of cliques to build places and networks of places instead of Galois lattices.

What we do here is:

1. Extract from the one-mode network all the cliques, using the standard clique definition[3] and the usual algorithms and programs to identify them.
2. Use the initial set of cliques to build a network of places.
3. Analyse the networks of places

This procedure allows us to find sets of structurally equivalent individuals and, also, to identify actors completely individualized by their network position in one-mode networks.

Then, we use either one- or two-mode networks in the following examples of the application of the concepts of places and networks of places.[4] The first two examples are 'classical' data sets of one-mode networks, used very often in social networks analysis articles. The two others come from our own empirical research, and are much bigger than the former.

## The 'Classical' Data Sets: Finding Non-Overlapping Groups

The data sets used by Freeman (1996) are two adjacency matrices, the first one coming from data collected by Roethlisberger and Dixon (1939) among

***Table 1*** *Data on Playing Games in the Bank Wiring Room*

| | I1 | 13 | W1 | W2 | W3 | W4 | W5 | W6 | W7 | W8 | W9 | S1 | S2 | S4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| I1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| W1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| W2 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| W3 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| W4 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| W5 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| W6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| W7 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| W8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| W9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| Si | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| S2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| S4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |

some employees in a Western Electric Company factory. The subjects are 14 men who worked in a 'Bank Wiring Room' and the adjacency matrix here is limited to the relation 'playing games', selected by Freeman as the more objective of the different relations studied by the authors (see Table 1).

The analysis of the relations gave five cliques, here expressed in the incidence matrix shown in Table 2.

***Table 2*** *Cliques Formed by Game Playing in the Bank Wiring Room*

| | C1 | C2 | C3 | C4 | C5 |
|---|---|---|---|---|---|
| I1 | 1 | 0 | 0 | 0 | 0 |
| I3 | 0 | 0 | 0 | 0 | 0 |
| W1 | 1 | 1 | 1 | 0 | 0 |
| W2 | 1 | 1 | 0 | 0 | 0 |
| W3 | 1 | 1 | 1 | 0 | 0 |
| W4 | 1 | 1 | 1 | 0 | 0 |
| W5 | 0 | 0 | 1 | 0 | 0 |
| W6 | 0 | 0 | 0 | 1 | 0 |
| W7 | 0 | 0 | 0 | 1 | 1 |
| W8 | 0 | 0 | 0 | 1 | 1 |
| W9 | 0 | 0 | 0 | 1 | 1 |
| S1 | 0 | 1 | 1 | 0 | 0 |
| S2 | 0 | 0 | 0 | 0 | 0 |
| S4 | 0 | 0 | 0 | 0 | 1 |

The Galois lattice shown in Figure 2 is the basis for the group's construction.

The two groups are: (S4, W6, W7, W8, W9) and (S1, I1, W1, W2, W3, W4, W5).

We found the following places:

P1 = {'c1'} 1 {'I1'} 1
P2 = {} 0 {'I3', 'S2'} 2 (The two persons not in the cliques. They appear here because we did not suppress their corresponding lines in the incidence matrix.)
P3 = {'c1', 'c2', 'c3'} 3 {'W1', 'W3', 'W4'} 3
P4 = {'c1', 'c2'} 2 {'W2'} 1
P5 = {'c3'} 1 {'W5'} 1
P6 = {'c4'} 1 {'W6'} 1
P7 = {'c4', 'c5'} 2 {'W7', 'W8', 'W9'} 3
P8 = {'c2', 'c3'} 2 {'S1'} 1
P9 = {'c5'} 1 {'S4'} 1

It is interesting to see that two of the places (P3 and P7) include three structurally equivalent persons each, and that all other places completely identify one individual each.

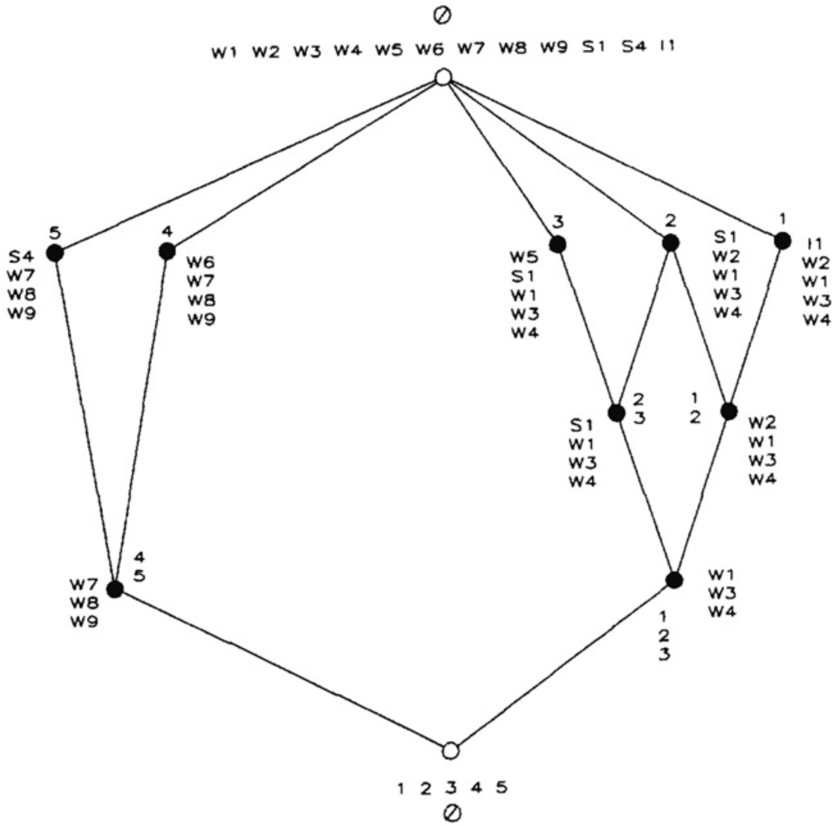



***Figure 2*** *Galois Lattice Representation of the Bank Wiring Room Data*
*Note:* Reproduced from Freeman (1996).

Also, it is worth noting that the 'depth' of individuals in Galois lattices defined by Freeman corresponds to the number of social circles that define each place.

The network of places: places connected by cliques is presented in Figure 3.

This network has two connected components. The individual's members of the places in each component correspond exactly with the groups identified by Freeman.

Here we have two components (Figure 4).

## The 'Classical' Data Sets: Finding Overlapping Groups

The second set used by Freeman (1996) comes from data collected by Sampson (1968) in a monastery, using only the adjacency matrix of the 'liking' relation between monks (Table 3).

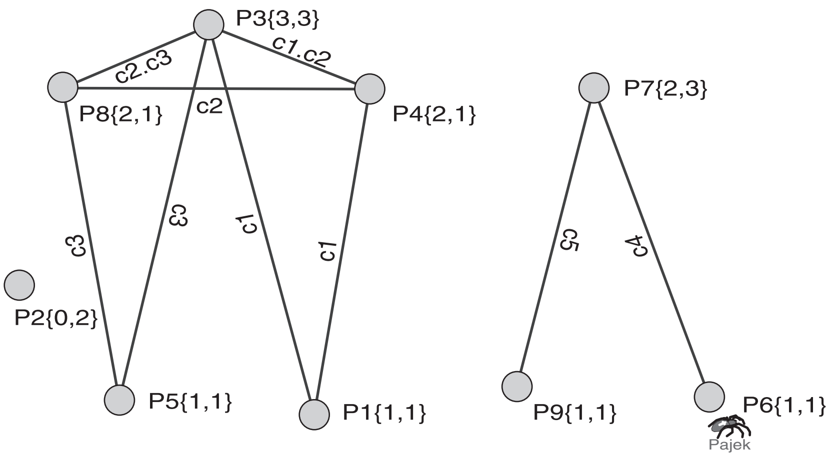



*Figure 3* Network of Places from Bank Wiring Room Data: Places Connected by Cliques

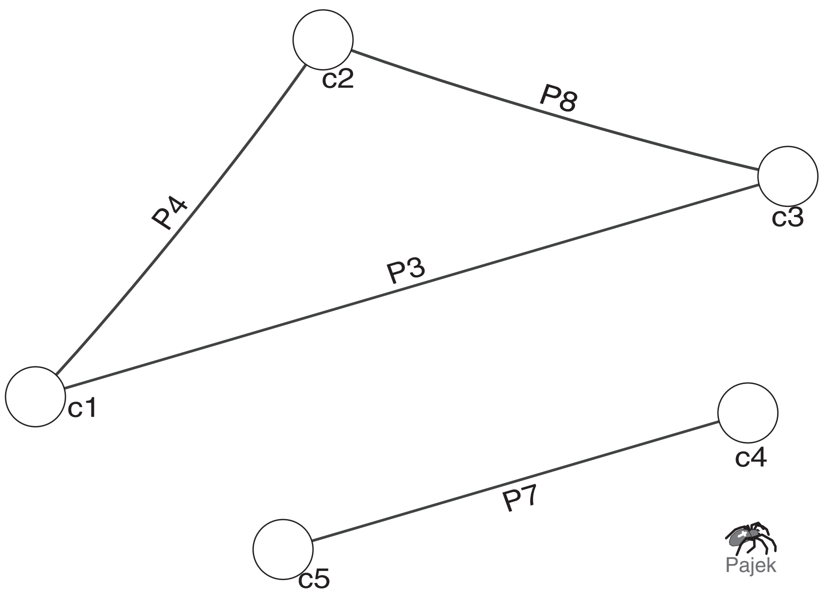



*Figure 4* The Dual Network: Cliques Connected by Places

Submitting these data to a clique analysis, we found the 17 cliques expressed in the incidence matrix shown in Table 4.

***Table 3*** *Sampson's (1968) Data on Reported Liking (Last Ranking)*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 2 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 4 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 8 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 12 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 14 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 15 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 17 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 18 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |

***Table 4*** *Cliques Formed by 'Liking' in the Monastery*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| m1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| m2 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| m3 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| m4 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| m5 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| m6 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| m7 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| m8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| m9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| m10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| m11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| m12 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| m13 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| m14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| m15 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| m16 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| m17 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| m18 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

The Galois lattice built by Freeman is presented in Figure 5. As we can see, it has 43 nodes.

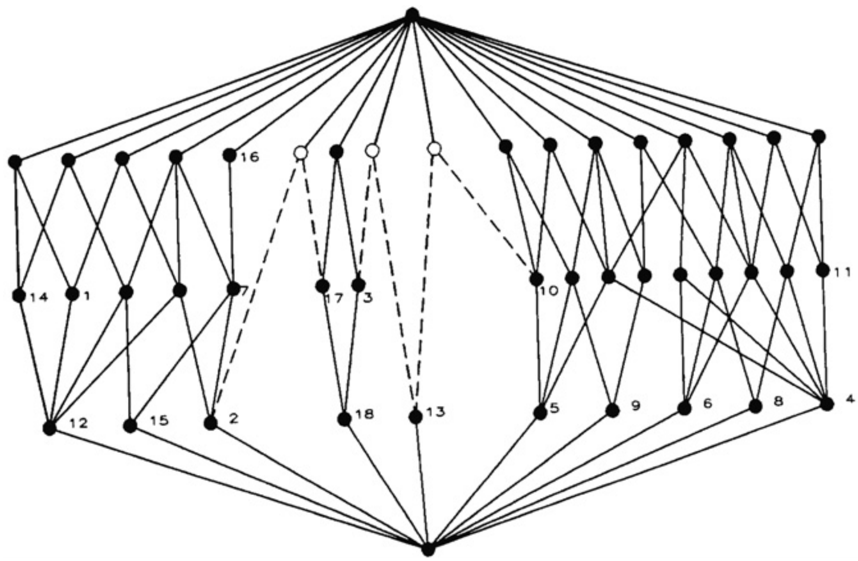



***Figure 5*** *Galois Lattice Representation of the Sampson's Data Set, from Freeman (1996)*

The cliques are the following:

1: 'm2' 'm7' m12' m15'
2: m2' m7' m15' m16'
3: m1' m2' m12'
4: m2' m17' m18'
5: m3' m13' m18'
6: m3' m17' m18'
7: m4' m5' m6'
8: m4' m5' m10'
9: m4' m5' m11'
10: m4' m6' m8'
11: m4' m8' m11'
12: m5' m10' m13'
13: m5' m9' m10'
14: m5' m6' m9'
15: m6' m8' m9'
16: m1' m12' m14'

17: m12' m14' m15'

We found the following places:

P1 = {c7, c8, c9, c12, c13, c14} 6 {m5′} 1
P2 = {c7, c8, c9, c10, c11} 5 {m4′} 1
P3 = {c1, c2, c3, c4} 4 {m2′} 1
P4 = {c7, c10, c14, c15} 4 {m6′} 1
P5 = {c1, c3, c16, c17} 4 {m12′} 1
P6 = {c10, c11, c15} 3 {m8′} 1
P7 = {c13, c14, c15} 3 {m9′} 1
P8 = {c8, c12, c13} 3 {m10′} 1
P9 = {c1,c2,c17} 3 {m15′} 1
P10 = {c4, c5, c6} 3 {m18′} 1
P11 = {c3, c16} 2 {m1′} 1
P12 = {c5,c6} 2 {m3′} 1
P13 = {c1, c2} 2 {m7′} 1
P14 = {c9, c11} 2 {m11′} 1
P15 = {c5,c12} 2 {m13′} 1
P16 = {c16, c17} 2 {m14′} 1
P17 = {c4, c6} 2 {m17′} 1
P18 = {c2} 1 {m16′} 1

It is interesting to note that all the places are occupied by one person each. That is to say, in this network social relations completely identify all the individuals.

The Galois lattice has 43 nodes (Figure 5), while the network of places has only 18 (Figure 6).

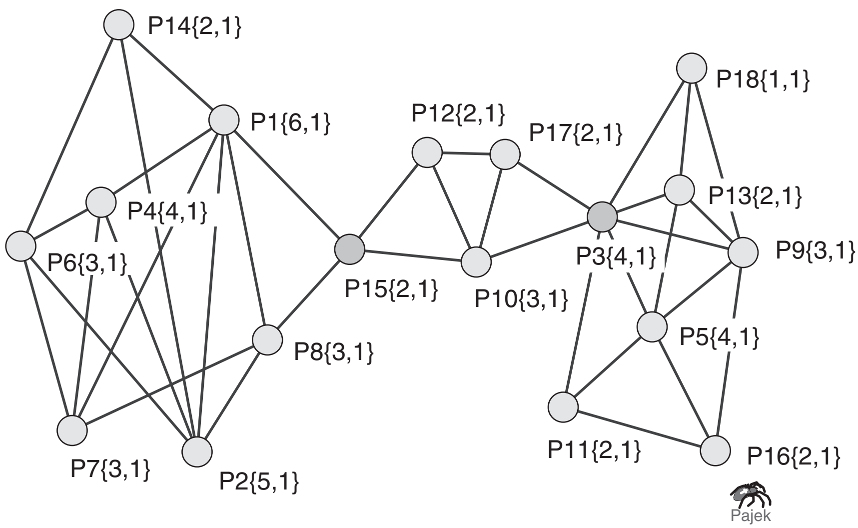



***Figure 6*** *Network of Places from Sampson's Data Set*

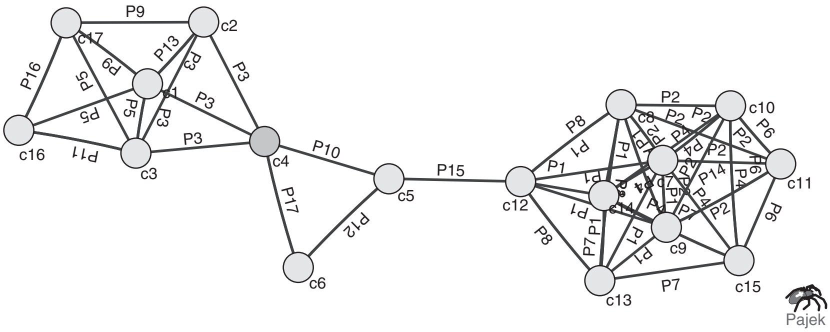



*Figure 7* *Sampson's Data Set: Dual Network of Cliques Connected by Places*

This network of places is connected, and so there are no components. But, looking at the bi-components, there are two cutting points, P3 and P15. We have then, three bi-components:

1: {P3(4,1), P5(4,1), P9(3,1), P13(2,1), P18(1,1), P16(2,1), P11(2,1)}
2: {P10(3,1), P15(2,1), P12(2,1), P17(2,1), P3(4,1)}
3: {P1(6,1), P2(5,1), P14(2,1), P4(4,1), P6(3,1), P7(3,1), P8(3,1), P15(2,1)}

The individuals corresponding to these bi-components of network of places are:

1: {m2, m12, m15, m7, m16, m14, m1}
2: {m18, m13, m3, m17, m2}
3: {m5, m4, m11, m6, m8, m9, m10, m13}

Figure 7 presents the dual network of cliques connected by places.

As in the prior case, we have here a connected network. Looking at the bi-components, we find:

1: {c7, c14, c12, c8, c13, c15, c9, c11, c10}
2: {c4, c5, c6}
3: {c1, c3, c2, c4, c16, c17}

## Another Case of Interlocking Social Circles: Associations in a Small Village in Extremadura

This case is more interesting, because the number of individuals is greater than in the two preceding networks, as well as greater than in the network employed by Falzon (2000). Data come from the boards of the 17 different types of associations we found in Logrosan, a small village in Extremadura (Spain) in 2004. The total number of individuals on these boards is 194.

We found 43 places:

P1 = {d09m,15p} 2 {7} 1
P2 = {d16d} 1 {31, 171, 584, 624, 685, 2118, 2210} 7
P3 = {d09m} 1 {36, 113, 373, 645, 831, 1091, 1359, 1408, 1436, 1632, 1728, 1958, 2148, 2200, 2223, 2260} 16
P4 = {04Pc} 1 {46,110, 249, 366, 592, 839, 975, 1029, 1077, 1151, 1244, 1301, 1370, 1573, 1814, 1874, 1908, 2096} 18
P5 = {13p} 1 {54, 100, 631, 899, 933, 934, 1250, 1457, 1509, 1647, 1684, 1694, 1947, 1952, 1966, 2015, 2017, 2083, 2119, 2172, 2253, 2254} 22
P6 = {d21n} 1 {60, 980, 1842, 2203} 4
P7 = {d17g} 1 {67, 286, 294, 563, 841} 5
P8 = {d01a} 1 {80, 427, 609, 970, 1418, 1689, 2073} 7
P9 = {12p} 1 {82, 821, 974, 1000, 1001, 1153, 1224, 1284,1299, 1402, 1413, 2060, 2061} 13
P10 = {14p} 1 {85, 111, 562, 680, 1016, 1076, 1201, 1586, 1588, 1615, 1619, 1962, 1986, 2109, 2255, 2256, 2257} 17
P11 = {15p} 1 {148, 442, 498, 506, 751, 759, 927, 1057, 1062, 1109, 1468, 1529, 1540, 1798, 1893, 2091, 2151, 2214, 2247, 2258, 2259} 21
P12 = {d02m} 1 {155, 264, 772, 1030, 1368, 1448, 1461} 7
P13 = {04Pc, d21n} 2 {409} 1
P14 = {d09m, d05r} 2 {432} 1
P15 = {15p, d08d} 2 {517} 1
P16 = {d11m} 1 {602, 1667} 2
P17 = {d09m, 14p} 2 {644, 2228} 2
P18 = {d09m, 12p} 2 {753} 1
P19 = {d08d} 1 {768, 1055, 1391, 1825, 2248} 5
P20 = {d21n, d05r} 2 {866} 1
P21 = {d18k} 1 {872} 1
P22 = {d17g, 14p} 2 {878} 1
P23 = {d07f} 1 {892, 1582} 2
P24 = {d05r} 1 {938, 1102, 1429, 1596, 1597, 1601, 1603, 1762, 1806, 2265, 2266} 11
P25 = {d04e} 1 {944, 1296, 1794, 1949, 2262, 2263, 2264} 7
P26 = {d16d, 13p} 2 {983} 1
P27 = {d09m, d17g, d07f} 3 {1017} 1
P28 = {12p, d05r} 2 {1087, 1604} 2
P29 = {14p, d11m} 2 {1132} 1
P30 = {d11m, d04e} 2 {1174} 1
P31 = {04Pc, 13p, d07f} 3 {1347} 1
P32 = {13p, d08d} 2 {1362} 1
P33 = {04Pc, 14p} 2 {1427, 1873} 2
P34 = {d21n, 12p} 2 {1440} 1
P35 = {04Pc, d05r} 2 {1449} 1

P36 = {d17g, d05r} 2 {1450} 1
P37 = {04Pc, d08d} 2 {1561} 1
P38 = {d16d, 12p} 2 {1676} 1
P39 = {d09m, d21n, 14p} 3 {1709} 1
P40 = {04Pc, d08d, d07f} 3 {1816} 1
P41 = {13p, d21n} 2 {2014} 1
P42 = {d09m,04Pc} 2 {2065} 1
P43 = {04Pc, d07f} 2 {2261} 1

Places are distributed by number of social circles and number of individuals, as shown in Table 5.

***Table 5***

| | | Number of individuals occupying the place | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | >2 | Total |
| Number of social circles per place | 1 | 1 | 2 | 13 | 16 |
| | 2 | 19 | 3 | 0 | 22 |
| | 3 | 5 | 0 | 0 | 5 |
| | Total | 25 | 5 | 13 | 43 |

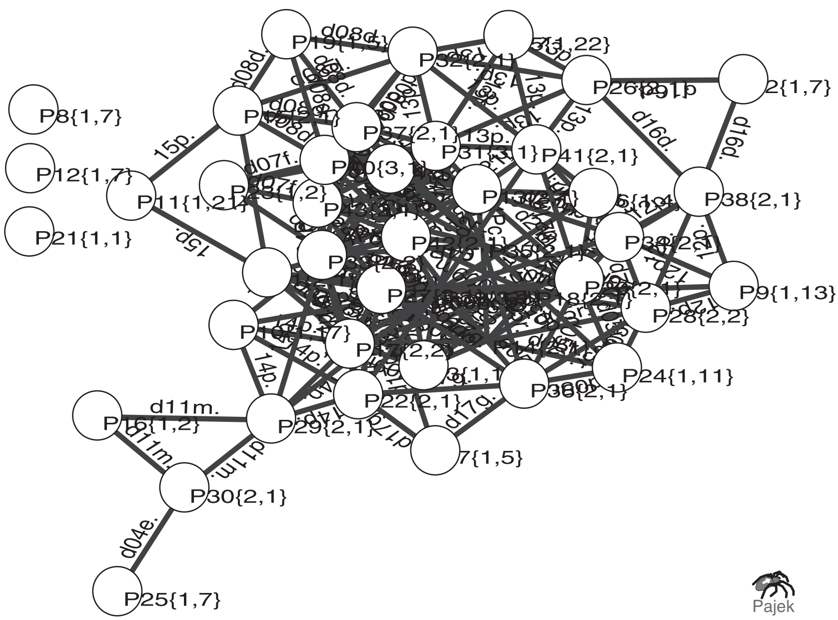



***Figure 8*** *Network of Places of Logrosan's Data Set*

Then, 25 individuals out of the 194 are completely identified by their position in the network and 169 occupy the remaining 18 places (classes of equivalent individuals).

The network of places is shown in Figure 8.

We see two bi-components and three isolated places. Place 29 is the cutting point between the bi-components.

## Social Circles in the Spanish Power Elite (1976–81)

The last and more interesting example is taken from our data about Spanish power elite in the Transition period (1976–81). The interest, from the point of view developed here, comes from the large size of the dual network we are considering.

***Table 6*** *Number of Places by Persons * Circles*

| Persons | Circles 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | Total places |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 83 | 1831 | 643 | 274 | 188 | 77 | 66 | 35 | 28 | 10 | 7 | 5 | 2 | 2 | **3251** |
| **2** | 143 | 90 | 2 | | | | | | | | | | | | **235** |
| **3** | 202 | 13 | | | | | | | | | | | | | **215** |
| **4** | 211 | 6 | | | | | | | | | | | | | **217** |
| **5** | 208 | 5 | | | | | | | | | | | | | **213** |
| **6** | 180 | 1 | | | | | | | | | | | | | **181** |
| **7** | 161 | | | | | | | | | | | | | | **161** |
| **8** | 99 | | | | | | | | | | | | | | **99** |
| **9** | 67 | 1 | | | | | | | | | | | | | **68** |
| **10** | 39 | | | | | | | | | | | | | | **39** |
| **11** | 18 | | | | | | | | | | | | | | **18** |
| **12** | 11 | | | | | | | | | | | | | | **11** |
| **13** | 7 | | | | | | | | | | | | | | **7** |
| **14** | 4 | | | | | | | | | | | | | | **4** |
| **15** | 1 | | | | | | | | | | | | | | **1** |
| **16** | 1 | | | | | | | | | | | | | | **1** |
| **17** | 1 | | | | | | | | | | | | | | **1** |
| **18** | 1 | | | | | | | | | | | | | | **1** |
| **19** | 1 | | | | | | | | | | | | | | **1** |
| **20** | 1 | | | | | | | | | | | | | | **1** |
| **21** | 2 | | | | | | | | | | | | | | **2** |
| **25** | 1 | | | | | | | | | | | | | | **1** |
| **28** | 2 | | | | | | | | | | | | | | **2** |
| **Total** | **1444** | **1947** | **645** | **274** | **188** | **77** | **66** | **35** | **28** | **10** | **7** | **5** | **2** | **2** | **4730** |

In this database, we have 11,014 individuals and 1484 social circles. The circles are administration boards of the biggest Spanish firms, political parties in the Congress and the Senate, corporations of high level civil servants and tenants of power positions in the public administration.

We found 4730 places out of the original dual network. As the list is too long, it is not included here and we summarize the results as follows.

The dual network of 12,498 nodes is reduced to 4730 places. Out of this total, 3251 places are occupied by a unique individual and so, completely identify the individual. The remaining 1479 places are occupied by two or more individuals, and so, they are classes of equivalent individuals in which 10,535 individuals are embedded.

## Conclusions

Places and networks of places are tools for the analysis of network structure that dissolve the duality of individuals and groups and that reduce the size of networks, because places are sets of structurally equivalent individuals, and individuals themselves are defined by the intersection of their memberships in social circles.

Moreover, the computation procedures needed to build networks of places are very simple and computation time is very short, even for large networks, precisely where reduction is required.

Networks of places allow us to consider the relational nature of the social identity of individuals, as defined by the intersection of social circles in the sense of Simmel (Breiger, 2000) and to identify classes of equivalent individuals. We have also used them to identify social groups.

When a place is occupied by a single individual, we can say that this individual is completely identified by the particular intersection of social circles that define the place. And when it is occupied by two or more individuals, they are structurally equivalent, which means that social circles cannot individualize more persons in the structure.

## Notes

1. The following exposition on duality follows, almost word for word, the Breiger (1974) article. We include it here to facilitate reading for the non-social-network analyst.
2. The study of indirect relations in networks of places is published in Pizarro (2004).
3. Cliques are defined as maximal complete subgraphs (Luce and Perry, 1949). The use of clique analysis in social network groups identification has been limited by the fact that cliques usually found are too small and too numerous, and

also by the fact that they overlap, having common members [ ]. Because of the existence of overlaps, it is normally impossible to use cliques to partition the network into two or more non-overlapping social groups.

4. We use a little program we wrote ourselves, called 'Places', available upon request.

## References

Breiger, R. L. (1974) 'The Duality of Persons and Groups', *Social Forces* 53(2): 181–90.

Breiger, R. L. (1990) 'Social Control and Social Networks: A Model from Georg Simmel', in C. Calhoun, M. W. Meyer and W. R. Scott (eds) *Structures of Power and Constraint: Papers in Honor of Peter M. Blau*, pp. 453–76. Cambridge: Cambridge University Press.

Breiger, R. L. (2000) 'Control social y redes sociales: un modelo a partir de Georg Simmel', *Política y Sociedad* 33: 57–72.

Doreian, P., Batagelj, V. and Ferligoj, A. (2004) 'Generalized Blockmodeling of Two-Mode Network Data', *Social Networks* 26: 29–53.

Falzon, L. (2000) 'Determining Groups from the Clique Structure in Large Social Networks', *Social Networks* 22(2): 159–72.

Freeman, L. C. (1996) 'Cliques, Galois Lattices, and the Structure of Human Social Groups', *Social Networks* 18: 173–87.

Lorrain, F. and White, H. C. (1971) 'Structural Equivalence of Individuals in Social Networks', *Journal of Mathematical Sociology* 1: 49–80.

Luce, R. D. and Perry, A. (1949) 'A Method of Matrix Analysis of Group Structure', *Psychometrika* 14: 95–116.

Nadel, S. F. (1956) *The Theory of Social Structure*. Cambridge: Cambridge University Press.

Pizarro, N. (2004) 'Un nuevo enfoque sobre la equivalencia estructural: lugares y redes de lugares como herramientas para la teoría sociológica', *Redes. Revista hispana para el análisis de redes sociales,* enero–febrero.

Roethlisberger, F. J. and Dixon, D. J. (1939) *Management and the Worker.* Cambridge, MA: Harvard University Press.

Sampson, S. F. (1968) 'A Noviate in a Period of Change: An Experimental Case Study of Social Relationships', PhD dissertation, Cornell University.

Simmel, Georg (1927) *Soziologie: Untersunchungen über die Formen der Vergesellschaftung* (Spanish trans., *Sociología. Un estudio sobre las formas de socialización*. Madrid: Revista de Occidente.)

White, H., Boorman, S. C. and Breiger, R. L. (1976) 'Social Structure from Multiple Networks. I. Blockmodels of Roles and Positions', *American Journal of Sociology* 81(4): 730–80.

***Biographical Note:*** Narciso Pizarro is Professor of Political and Administrative Science, Universidad Complutense de Madrid, and formerly of the Université du Québec à Montréal. His research is in the areas of social networks, power elites and social structure theory.

***Address:*** Departamento de Ciencia Política y de la Administración (II), Facultad de Ciencias Políticas y Sociología, Universidad Complutense de Madrid, Campus de Somosaguas s/n, Madrid 28223, Spain. [email: npizarro@cps.ucm.es]